2.3 Public Infrastructure Model

$$\max_{K,L} \quad \pi = f(K,L) - rK - c(K,W)K - \tau K - wL$$

$$\begin{aligned} f_K &= r + c(K,W) + \tau \\ f_L &= \omega \end{aligned}$$

$$\tau^L L + \tau K = \rho W$$

$$\begin{aligned} &\max_{K,W} \quad R = \omega L + r\bar{K} - \tau^L L \\ &\max_{K,W} \quad R = f(K,L) - f_K K + r\bar{K} - \tau^L L \\ &\max_{K,W} \quad R = f(K,L) + r(\overline{K} - K) - c(K,W)K - \rho W \end{aligned}$$

$$\begin{aligned} f_K &= r + c(K,W) + c_K K \\ -c_W K &= \rho \end{aligned}$$

$$\tau = c_K K$$

$$\max_{K_i,W_i} \quad \sum_{i=1}^{n} [f(K_i,L_i) - c(K_i,W_i)K_i - \rho W_i]$$

$$\text{s.t.} \quad \sum_{i=1}^{n} K_i = \sum_{i=1}^{n} \bar{K}_i$$

$$\begin{aligned} f_{K_i} - c(K_i,W_i) - c_{K_i}K_i &= f_{K_j} - c(K_j,W_j) - c_{K_j}K_j && \forall i \neq j \\ -c_W K_i &= \rho && \forall i \end{aligned}$$

$$c_K K + c_W W = \lambda c(K,W)$$

$$\begin{aligned} -c_W K = \rho \quad &\Rightarrow \quad c_W = -\frac{\rho}{K} \\ \tau = c_K K \quad &\Rightarrow \quad c_K = \frac{\tau}{K} \end{aligned}$$

$$-\frac{\tau}{K}K + \frac{\rho}{K}W = \lambda c(K,W)$$

$$\tau K = \rho W + \lambda c(K,W)K$$