2.1 Public Good Model

$$\max_{k,l} \quad \pi = f(k,l) - (r+\tau)k - wl$$

$$f_k = r + \tau$$
$$f_l = \omega$$

$$f_{kk}^i dk^i = \left(\frac{\partial r}{\partial \tau^i} d\tau^i + d\tau^i\right) \qquad f_{kk}^j dk^j = \frac{\partial r}{\partial \tau^i} d\tau^i$$

$$\frac{dk^i}{d\tau^i} = \frac{\frac{\partial r}{\partial \tau^i} + 1}{f_{kk}^i} \qquad \frac{dk^j}{d\tau^i} = \frac{\partial r}{\partial \tau^i} \frac{1}{f_{kk}^j}$$

$$n\bar{k} = \sum_{i=1}^{n} k^i$$
$$0 = \frac{\partial k^i}{\partial \tau^i} + \sum_{j=1, j\neq i}^{n} \frac{\partial k^j}{\partial \tau^i}$$
$$0 = \frac{\partial k^i}{\partial \tau^i} + (n-1)\frac{\partial k^j}{\partial \tau^i}$$
$$0 = \frac{\frac{\partial r}{\partial \tau^i} + 1}{f_{kk}^i} + (n-1)\frac{\partial r}{\partial \tau^i}\frac{1}{f_{kk}^j}$$
$$\frac{\partial r}{\partial \tau^i} = -\frac{1}{n}$$

$$\frac{\partial k^i}{\partial \tau^i} = \frac{n-1}{n}\frac{1}{f_{kk}^i} \qquad \frac{\partial k^j}{\partial \tau^i} = -\frac{1}{n}\frac{1}{f_{kk}^j}$$

$$c = wl + r\bar{k}$$
$$c = f(k) - f_k k + r\bar{k} \qquad g = \tau k$$

$$\max_{\tau} \quad u\left[c, g\right]$$

$$\max_{\tau} \quad u\left[f(k) - f_k k + r\bar{k},\, \tau k\right]$$

$$\frac{du}{d\tau} = u_c\left(-f_{kk}\frac{\partial k}{\partial \tau}k + \frac{\partial r}{\partial \tau}\bar{k}\right) + u_g\left(k + \tau\frac{\partial k}{\partial \tau}\right) \stackrel{!}{=} 0$$
$$u_c\left[\left(-f_{kk}\frac{\partial k}{\partial \tau} + \frac{\partial r}{\partial \tau}\right)k\right] + u_g\left(k + \tau\frac{\partial k}{\partial \tau}\right) = 0$$
$$u_c\left(-k\right) + u_g\left(k + \tau\frac{\partial k}{\partial \tau}\right) = 0$$

$$\frac{u_g}{u_c} = \frac{1}{1 + \frac{\partial k}{\partial \tau}\frac{\tau}{k}}$$

$$u_g = u_c\frac{1}{1 + \frac{\partial k}{\partial \tau}\frac{\tau}{k}}$$