

5.1 Spill-over

$$\max_{L_i, S_i, K_i} f(L_i, S_i, K_i) - wL_i - pS_i - rK_i$$

$$f_L = w \quad f_S = p \quad f_K = r$$

$$T_i = pS_i$$

$$S_i^* = \frac{1}{n} \sum_{j=1}^n S_j \quad \forall i = 1, \dots, n$$

$$\begin{aligned} f(L_i, S_i, K_i) &= f_L L_i + f_S S_i + f_K K_i \\ wL_i + pS_i &= f(L_i, S_i, K_i) - f_K K_i \end{aligned}$$

$$Y_i \equiv wL_i + T_i + r\bar{K}$$

$$Y_i = wL_i + pS_i + r\bar{K}$$

$$Y_i = f(L_i, S_i, K_i) - f_K K_i + r\bar{K}$$

$$Y_i = f(L_i, S_i, K_i) - r(\bar{K} - K_i)$$

$$\max_{S_i} U(Y_i, S_i^*)$$

$$\max_{S_i} U \left(f(L_i, S_i, K_i) + r(\bar{K} - K_i), \frac{1}{n} \sum_{j=1}^n S_j \right)$$

$$\frac{\partial U_i}{\partial S_i} = U_Y \left(f_S + f_K \frac{\partial K_i}{\partial S_i} - r \frac{\partial K_i}{\partial S_i} \right) + \frac{1}{n} U_S \stackrel{!}{=} 0$$

$$U_Y \cdot f_S = -\frac{1}{n} U_S$$

$$f_S = -\frac{1}{n} \frac{U_S}{U_Y}$$