3.3 Thum and Uebelmesser (2003)

$$Y_i = F_i(L_i) \quad \text{with} \quad \frac{\partial F_i}{\partial L_i} = m_i$$

$$L_i = I_i N_i + I_j^i N_j^i$$

$$I_i = \gamma_i Z_i$$

$$w_i = m_i \gamma_i Z_i (1 - t_i)$$
$$w_i^j = m_j (1 - \gamma_i) Z_i (1 - t_i^j)$$

$$t_i^j = 0$$

$$w_i = w_i^j$$
$$m_i \gamma_i Z_i (1 - t_i) = m_j (1 - \gamma_i) Z_i$$

$$t_i^* = 1 - \frac{m_j (1 - \gamma_i)}{m_i \gamma_i}$$

$$\max_{Z_i} \quad m_i \gamma_i Z_i (1 - t_i) - Z_i^2$$

$$\frac{d}{dZ_i} = m_i \gamma_i (1 - t_i) - 2 Z_i \stackrel{!}{=} 0$$
$$\frac{m_i \gamma_i m_j (1 - \gamma_i)}{m_i \gamma_i} = 2 Z_i$$
$$Z_i^* = \frac{1}{2} m_j (1 - \gamma_i)$$

$$\max_{\gamma_i} \quad m_i \gamma_i Z_i^* t_i^*$$
$$\max_{\gamma_i} \quad m_i \gamma_i \frac{1}{2} m_j \left(1 - \gamma_i\right) \left(1 - \frac{m_j \left(1 - \gamma_i\right)}{m_i \gamma_i}\right)$$

$$\frac{d}{d\gamma_i} = [\ldots\ldots + \ldots\ldots] + [\ldots\ldots\ldots\ldots] \stackrel{!}{=} 0$$
$$m_i m_j \left(\frac{1}{2} - \gamma_i\right) = -m_j^2 \left(1 - \gamma_i\right)$$

$$\gamma_i^* = \frac{1}{2} \frac{m_i + 2m_j}{m_i + m_j}$$

$$Z_i^* = \frac{1}{4} \frac{m_i m_j}{m_i + m_j} \qquad\qquad t_i^* = \frac{m_i + m_j}{m_i + 2m_j}$$