

2.3 Public Infrastructure Model

$$\max_{K,W} \quad \pi = f(K, L) - rK - c(K, W)K - \tau K - wL$$

$$f_K = r + c(K, W) + \tau$$

$$f_L = \omega$$

$$\tau^L L + \tau K = \rho W$$

$$\max_{K,W} \quad R = \omega L + r\bar{K} - \tau^L L$$

$$\max_{K,W} \quad R = f(K, L) - f_K K + r\bar{K} - \tau^L L$$

$$\max_{K,W} \quad R = f(K, L) + r(\bar{K} - K) - c(K, W)K - \rho W$$

$$f_K = r + c(K, W) + c_K K$$

$$-c_W K = \rho$$

$$\tau = c_K K$$

$$\max_{K_i, W_i} \quad \sum_{i=1}^n [f(K_i, L_i) - c(K_i, W_i)K_i - \rho W_i]$$

$$\text{s.t.} \quad \sum_{i=1}^n K_i = \sum_{i=1}^n \bar{K}_i$$

$$f_{K_i} - c(K_i, W_i) - c_{K_i} K_i = f_{K_j} - c(K_j, W_j) - c_{K_j} K_j \quad \forall i \neq j$$

$$-c_W K_i = \rho \quad \forall i$$

$$c_K K + c_W W = \lambda c(K, W)$$

$$-c_W(K, W)K = \rho \quad \Rightarrow \quad c_W(K, W) = -\frac{\rho}{K}$$

$$\tau = c_K(K, W)K \quad \Rightarrow \quad c_K(K, W) = \frac{\tau}{K}$$

$$-\frac{\rho}{K}K + \frac{\tau}{K}W = \lambda c(K, W)$$

$$\tau K = \rho W + \lambda c(K, W)K$$