

During the discussion of the model of Thum and Uebelmesser (2003) there was confusion about two questions. I hope that I can iron them out here.

How does the result for the degree of international applicability depend on productivities?

The central result of the model is that the government does not set the degree of international applicability γ_i to 1, but to a lower value. The solution for the optimal γ_i is given by equation (1).

There was no disagreement about this result during the tutorial. However, I did not quite remember how to rearrange the result to easily see the effects of changes in m_i and m_j on γ_i^* . Equation (1) can be rearranged into something that is easier to interpret. This is done by splitting up $2m_j$ into $m_j + m_j$. One m_j goes onto a second fraction. $(m_i + m_j)$ in the first fraction cancels and we arrive at equation (3).

$$\gamma_i^* = \frac{1}{2} \frac{m_i + 2m_j}{m_i + m_j} \tag{1}$$

$$\gamma_i^* = \frac{m_i + m_j}{2 \cdot (m_i + m_j)} + \frac{m_j}{2 \cdot (m_i + m_j)} \tag{2}$$

$$\gamma_i^* = \frac{1}{2} + \frac{m_j}{2m_i + 2m_j} \tag{3}$$

Now we can easily see that γ_i^* decreases in m_i . It appears only once in equation (3). On the other hand, m_j still appears twice. However, since the denominator consists of the sum of $2m_j$ and another positive term, it will always increase more slowly than the numerator, which only consists of m_j .

If this explanation does not convince you, you can always treat the second fraction as a function, take its derivative with respect to m_j and determine its sign. A positive derivative implies that the fraction increases in m_j . Here is how to do that¹:

$$f(m_j) \equiv \underbrace{\frac{\overset{u}{m_j}}{2m_i + 2m_j}}_v \Rightarrow f'(m_j) = \frac{\underbrace{(2m_i + 2m_j)}_v \cdot \underbrace{1}_{u'} - \underbrace{2}_{v'} \cdot \underbrace{m_j}_u}{\underbrace{(2m_i + 2m_j)^2}_{v^2}} = \frac{2m_i}{(2m_i + 2m_j)^2} > 0$$

Does it matter whether the marginal productivities of countries are larger or smaller than 1?

The short answer to this question is no. The marginal productivities m_i and m_j have to be positive for the model to make sense, but it does not matter whether they are smaller or larger than one.

This result can be obtained in the same way as above: If you are uncertain about the effect of m_i or m_j in the results for Z_i^* and t_i^* , just treat the result as a function and take its derivative with respect to the marginal productivity in question. You should find that the sign of the derivatives is the same for values below and above 1. If you need help doing the calculations, feel free to approach me before or after the next tutorial.

References

Thum, Claudio and Silke Uebelmesser (2003). "Mobility and the Role of Education as a Commitment Device". In: *International Tax and Public Finance* 10.5 (2003), pp. 549–564.

¹I apply the quotient rule to calculate the derivative.