

3.3 Thum and Uebelmesser (2003)

$$Y_i = F_i(L_i) \quad \text{with} \quad \frac{\partial F_i}{\partial L_i} = m_i$$

$$L_i = I_i N_i + I_j^i N_j^i$$

$$I_i = \gamma_i Z_i$$

$$\begin{aligned} w_i &= m_i \gamma_i Z_i (1 - t_i) \\ w_i^j &= m_j (1 - \gamma_i) Z_i (1 - t_i^j) \end{aligned}$$

$$t_i^j = 0$$

$$\begin{aligned} w_i &= w_i^j \\ m_i \gamma_i Z_i (1 - t_i) &= m_j (1 - \gamma_i) Z_i \end{aligned}$$

$$t_i^* = 1 - \frac{m_j (1 - \gamma_i)}{m_i \gamma_i}$$

$$\max_{Z_i} \quad m_i \gamma_i Z_i (1 - t_i) - Z_i^2$$

$$\begin{aligned} \frac{d}{dZ_i} &= m_i \gamma_i (1 - t_i) - 2Z_i \stackrel{!}{=} 0 \\ \frac{m_i \gamma_i m_j (1 - \gamma_i)}{m_i \gamma_i} &= 2Z_i \\ Z_i^* &= \frac{1}{2} m_j (1 - \gamma_i) \end{aligned}$$

$$\begin{aligned} \max_{\gamma_i} \quad & m_i \gamma_i Z_i^* t_i^* \\ \max_{\gamma_i} \quad & m_i \gamma_i \frac{1}{2} m_j (1 - \gamma_i) \left(1 - \frac{m_j (1 - \gamma_i)}{m_i \gamma_i} \right) \end{aligned}$$

$$\begin{aligned} \frac{d}{d\gamma_i} &= [..... +] + [.....] \stackrel{!}{=} 0 \\ m_i m_j \left(\frac{1}{2} - \gamma_i \right) &= -m_j^2 (1 - \gamma_i) \end{aligned}$$

$$\gamma_i^* = \frac{1}{2} \frac{m_i + 2m_j}{m_i + m_j}$$

$$Z_i^* = \frac{1}{2} \frac{m_i m_j}{m_i + m_j} \quad t_i^* = \frac{m_i + m_j}{m_i + 2m_j}$$