

5.2 Optimal Pollution with Side Payments

$$\max_{L_i, S_i, K_i} f(L_i, S_i, K_i) - wL_i - pS_i - rK_i$$

$$f_L = w \quad f_S = p \quad f_K = r$$

$$T_i = pS_i$$

$$S_i^* = \frac{1}{n} \sum_{j=1}^n S_j \quad \forall i = 1, \dots, n$$

$$f(L_i, S_i, K_i) = f_L L_i + f_S S_i + f_K K_i$$

$$wL_i + pS_i = f(L_i, S_i, K_i) - f_K K_i$$

$$Y_i \equiv wL_i + T_i + r\bar{K} - \sum_{j=1, j \neq i}^n (Y_j - \bar{Y}_{ij})$$

$$Y_i = f(L_i, S_i, K_i) - r(\bar{K} - K_i) - \sum_{j=1, j \neq i}^n (Y_j - \bar{Y}_{ij})$$

$$Y_i = f(L_i, S_i, K_i) + r(\bar{K} - K_i) - S_i \sum_{j=1 \neq i}^n \frac{1}{n} \frac{dY_j}{dS_j^*}$$

$$\max_{S_i, Y_j} U(Y_i, S_i^*)$$

$$\max_{S_i, Y_j} U \left(f(L_i, S_i, K_i) + r(\bar{K} - K_i) - S_i \sum_{j=1 \neq i}^n \frac{1}{n} \frac{dY_j}{dS_j^*}, \frac{1}{n} \sum_{j=1}^n S_j \right)$$

$$\frac{\partial U_i}{\partial S_i} = U_{Y_i} \left(f_S - \sum_{j=1 \neq i}^n \frac{1}{n} \frac{dY_j}{dS_j^*} \right) + \frac{1}{n} U_{S_i} \stackrel{!}{=} 0$$

$$f_S - \sum_{j=1 \neq i}^n \frac{1}{n} \frac{dY_j}{dS_j^*} = \frac{1}{n} \left(-\frac{U_{S_i}}{U_{Y_i}} \right)$$

$$f_S - \sum_{j=1 \neq i}^n \frac{1}{n} \frac{dY_j}{dS_j^*} = \frac{1}{n} \frac{dY_i}{dS_i^*}$$

$$f_S = \sum_{j=1}^n \frac{1}{n} \frac{dY_j}{dS_j^*}$$

$$f_S = \frac{dY_j}{dS_j^*}$$