5.1 International Spillover

$$\max_{L_i,S_i,K_i} \quad f(L_i,S_i,K_i) - wL_i - pS_i - rK_i$$

$$f_L = w \qquad f_S = p \qquad f_K = r$$

$$T_i = pS_i$$

$$S_i^* = \frac{1}{n}\sum_{j=1}^{n} S_j \quad \forall i = 1,\dots,n$$

$$\begin{aligned} f(L_i,S_i,K_i) &= f_L L_i + f_S S_i + f_K K_i \\ wL_i + pS_i &= f(L_i,S_i,K_i) - f_K K_i \end{aligned}$$

$$Y_i \equiv wL_i + T_i + r\bar{K}$$

$$\begin{aligned} Y_i &= wL_i + pS_i + r\bar{K} \\ Y_i &= f(L_i,S_i,K_i) - f_K K_i + r\bar{K} \\ Y_i &= f(L_i,S_i,K_i) - r(\bar{K} - K_i) \end{aligned}$$

$$\max_{S_i} \quad U(Y_i, S_i^*)$$

$$\max_{S_i} \quad U\left(f(L_i,S_i,K_i) + r(\bar{K} - K_i), \frac{1}{n}\sum_{j=1}^{n} S_j\right)$$

$$\frac{\partial U_i}{\partial S_i} = U_Y\left(f_S + f_K\frac{\partial K_i}{\partial S_i} - r\frac{\partial K_i}{\partial S_i}\right) + \frac{1}{n}U_S \stackrel{!}{=} 0$$

$$U_Y \cdot f_S = -\frac{1}{n}U_S$$

$$f_S = -\frac{1}{n}\frac{U_S}{U_Y}$$