

2.1 Public Good Model

$$\max_{k,l} \quad \pi = f(k,l) - (r + \tau)k - \omega l$$

$$f_k = r + \tau$$

$$f_l = \omega$$

$$f_{kk}^i dk^i = \left(\frac{\partial r}{\partial \tau^i} d\tau^i + d\tau^i \right)$$

$$\frac{dk^i}{d\tau^i} = \frac{\frac{\partial r}{\partial \tau^i} + 1}{f_{kk}^i}$$

$$f_{kk}^j dk^j = \frac{\partial r}{\partial \tau^i} d\tau^i$$

$$\frac{dk^j}{d\tau^i} = \frac{\partial r}{\partial \tau^i} \frac{1}{f_{kk}^j}$$

$$n\bar{k} = \sum_{i=1}^n k^i$$

$$0 = \frac{\partial k^i}{\partial \tau^i} + \sum_{j=1, j \neq i}^n \frac{\partial k^j}{\partial \tau^i}$$

$$0 = \frac{\partial k^i}{\partial \tau^i} + (n-1) \frac{\partial k^j}{\partial \tau^i}$$

$$0 = \frac{\frac{\partial r}{\partial \tau^i} + 1}{f_{kk}^i} + (n-1) \frac{\partial r}{\partial \tau^i} \frac{1}{f_{kk}^j}$$

$$\frac{\partial r}{\partial \tau^i} = -\frac{1}{n}$$

$$\frac{\partial k^i}{\partial \tau^i} = \frac{n-1}{n} \frac{1}{f_{kk}^i}$$

$$\frac{\partial k^j}{\partial \tau^i} = -\frac{1}{n} \frac{1}{f_{kk}^j}$$

$$c = \omega l + r\bar{k}$$

$$c = f(k) - f_k k + r\bar{k}$$

$$g = \tau k$$

$$\max_{\tau} \quad u[c, g]$$

$$\max_{\tau} \quad u[f(k) - f_k k + r\bar{k}, \tau k]$$

$$\frac{du}{d\tau} = u_c \left(-f_{kk} \frac{\partial k}{\partial \tau} k + \frac{\partial r}{\partial \tau} \bar{k} \right) + u_g \left(k + \tau \frac{\partial k}{\partial \tau} \right) \stackrel{!}{=} 0$$

$$u_c \left[\left(-f_{kk} \frac{\partial k}{\partial \tau} + \frac{\partial r}{\partial \tau} \right) k \right] + u_g \left(k + \tau \frac{\partial k}{\partial \tau} \right) = 0$$

$$u_c (-k) + u_g \left(k + \tau \frac{\partial k}{\partial \tau} \right) = 0$$

$$\frac{u_g}{u_c} = \frac{1}{1 + \frac{\partial k}{\partial \tau} \frac{\tau}{k}}$$

$$u_g = u_c \frac{1}{1 + \frac{\partial k}{\partial \tau} \frac{\tau}{k}}$$